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**Preheating in an asymptotically safe quantum field theory**Ole Svendsen<sup>\*</sup>*CP<sup>3</sup>-Origins, University of Southern Denmark, Campusvej 55, DK-5230 Odense M, Denmark*Hossein Bazrafshan Moghaddam<sup>†</sup>*Department of Physics, McGill University, Montreal, Quebec H3A 2T8, Canada*Robert Brandenberger<sup>‡</sup>*Department of Physics, McGill University, Montreal, Quebec H3A 2T8, Canada  
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We consider reheating in a class of asymptotically safe quantum field theories recently studied in [D. F. Litim and F. Sannino, Asymptotic safety guaranteed, *J. High Energy Phys.* **12** (2014) 178; D. F. Litim, M. Mojaza, and F. Sannino, Vacuum stability of asymptotically safe gauge-Yukawa theories, *J. High Energy Phys.* **01** (2016) 081]. These theories allow for an inflationary phase in the very early universe. Inflation ends with a period of reheating. Since the models contain many scalar fields which are intrinsically coupled to the inflaton there is the possibility of parametric resonance instability in the production of these fields, and the danger that the induced curvature fluctuations will become too large. Here we show that the parametric instability indeed arises, and that hence the energy transfer from the inflaton condensate to fluctuating fields is rapid. Demanding that the curvature fluctuations induced by the parametrically amplified entropy modes do not exceed the upper observational bounds puts a lower bound on the number of fields which the model followed in [D. F. Litim and F. Sannino, Asymptotic safety guaranteed, *J. High Energy Phys.* **12** (2014) 178; D. F. Litim, M. Mojaza, and F. Sannino, Vacuum stability of asymptotically safe gauge-Yukawa theories, *J. High Energy Phys.* **01** (2016) 081] must contain. This bound also depends on the total number of  $e$ -foldings of the inflationary phase.

DOI: [10.1103/PhysRevD.94.083527](https://doi.org/10.1103/PhysRevD.94.083527)**I. INTRODUCTION**

Recently, interesting results have come up in the area of asymptotically safe quantum field theories [1,2], where it has been shown that a gauge-Yukawa theory can exhibit an ultraviolet (UV) safe (i.e. nontrivial) fixed point. Being UV safe these models could be interesting for early time cosmology. In particular, it has been realized that these models admit a period of cosmological inflation at very early times [3].

In order to connect an early inflationary phase with late time cosmology, a period of “reheating” at the end of inflation is required. During the reheating phase, the energy which is stored in coherent oscillations of the inflaton field at the end of the period of inflation is transferred to a thermal bath of Standard Model particles, yielding the onset of the postinflationary radiation phase of cosmological expansion. Reheating was initially studied using lowest order perturbation theory [4]. This process is typically slow and leads to a low initial temperature of the radiation phase.

However, as was realized in [5,6], there may be a parametric resonance instability which leads to a very

rapid transfer of energy from the inflaton condensate to fluctuations of the fields to which the inflaton couples. This initial phase of energy transfer is called “preheating” [7] and was studied in detail in [8,9]. (See e.g. [10] and [11] for recent reviews.)

The preheating process typically leads to a nonthermal state in which modes in certain wavelength intervals are highly excited whereas the rest are not excited at all. However, it leads to a phase in which the equation of state of matter is approximately that of radiation. For early universe considerations such as baryogenesis or the production of topological defects it is important to know the energy density when the radiation phase of expansion begins. Hence, it is important to know whether the preheating process is operative or not.<sup>1</sup> The first motivation for our study is to find out whether in the asymptotically safe quantum field models discussed in [3] preheating occurs.

If an inflationary universe model admits a preheating instability at the end of the period of inflation, there is the danger that the instability will also affect the cosmological perturbations [12]. The period of inflation produces fluctuations [13] which have the right spectral shape to explain the observed distribution of matter in the Universe and the

<sup>\*</sup>svendsen@cp3-origins.net<sup>†</sup>bazrafshan@physics.mcgill.ca<sup>‡</sup>rhb@physics.mcgill.ca<sup>1</sup>Preheating does not arise in all inflationary models.

observed cosmic microwave background anisotropies. A parametric growth of these fluctuations at the end of inflation would destroy the agreement between theory and observation. It was shown that in a model with only a single scalar matter field, there is no parametric amplification of the curvature fluctuations during reheating [14,15]. However, in the presence of a second scalar field a preheating instability of curvature fluctuations is possible [16,17]. If we denote the inflaton field by  $\varphi$  and the second scalar field by  $\chi$ , then it is found that if the  $\chi$  field experiences a parametric resonance instability due to the coupling to the inflaton field (which is oscillating at the end of inflation), then an entropy fluctuation is generated which leads to a growing curvature perturbation, even on length scales which are much larger than the Hubble radius.

It is important to point out why the growth of super-Hubble scale perturbations during reheating is compatible with causality. This was already discussed in [13]. The key point is that in inflationary cosmology there is an exponentially large difference between the horizon (the forward light cone of a point on the initial condition surface, e.g. a point at the beginning of inflation) which grows exponentially in time, and the Hubble radius, the inverse expansion rate (which is constant during a period of exponential inflation). The reason why inflation can provide a solution of the horizon problem of standard big bang cosmology is precisely the fact that the physical scale of a region of causal contact and homogeneity expands exponentially and becomes much larger than the Hubble radius. At the end of inflation, the inflaton field is in the coherent state and the correlation length of the field would set the maximum wavelength of fluctuations which can causally be amplified. Inflation will provide that the background inflaton field is coherent over a distance much larger than the Hubble radius during reheating and therefore guarantees the possibility of causal amplification of super-Hubble modes. This possibility is the case for both adiabatic and entropy fluctuations. Also we would like to mention the fact that field equations are relativistic, hence causality is mathematically built in and the result from field equations will not violate causality. In this regard there is no distinction between super-Hubble and sub-Hubble modes. For further details one may refer to Sec. V of [18].

The asymptotically safe quantum field models studied in [3] contains many scalar fields. A second goal of our study is to see whether there is a parametric amplification of entropy modes in our models, and what the resulting amplitude of the induced curvature fluctuations is.<sup>2</sup>

We find that there is indeed a parametric amplification of the fluctuations of the  $\chi$  fields in our model, and that this leads to induced curvature fluctuations which grow on super-Hubble scales during reheating. However, in the case

in which the model has a very large number of scalar fields (this is the limit in which calculations leading to the presence of the ultraviolet safe fixed point are under good control) we find that backreaction effects shut off the instability before the induced curvature fluctuations become too large.

## II. MODEL

We will here give a short recap of the model at hand. The full Lagrangian density is composed of adjoint  $SU(N_c)$  gauge fields,  $N_f$  Dirac fermions in the fundamental of  $SU(N_c)$  and an  $N_f \times N_f$  neutral complex scalar matrix,  $H$ . We will here only present the scalar part of the Lagrangian:

$$\mathcal{L} \supset \text{Tr}(\partial_\mu H^\dagger \partial^\mu H) - u \text{Tr}(H^\dagger H H^\dagger H) - v (\text{Tr} H^\dagger H)^2, \quad (1)$$

where  $u$  and  $v$  are dimensionless coupling constants.

As we will be working in the UV regime of this model it is noteworthy that the scalar couplings at the UV fixed point [1] are given by

$$\alpha_u^* = \frac{u^* N_f}{(4\pi)^2} = \frac{\sqrt{23} - 1}{19} \delta \quad (2)$$

$$\alpha_v^* = \frac{v^* N_f^2}{(4\pi)^2} = -\frac{1}{19} \left( 2\sqrt{23} - \sqrt{20 + 6\sqrt{23}} \right) \delta, \quad (3)$$

where the constant

$$\delta \equiv \frac{N_f}{N_c} - \frac{11}{2} \quad (4)$$

(which must be positive) can be made arbitrarily small by adjusting  $N_c$  and  $N_f$ .

We will in this work take a simplified version of (1) as we assume  $H$  to be symmetric and real. The parametrization of  $H$  is given by

$$H_{ij} = \begin{cases} \frac{1}{\sqrt{2N_f}} \phi & \text{if } i = j \\ \frac{1}{2} \chi_{(ij)} & \text{otherwise} \end{cases}, \quad (5)$$

where  $(ij)$  indicates that this part is symmetric in  $i$  and  $j$ .

With this normalization the kinetic term in (1) is given by

$$\text{Tr}(\partial_\mu H^\dagger \partial^\mu H) = \frac{1}{2} \partial_\mu \phi \partial^\mu \phi + \frac{1}{2} \sum_l^{N_p} \partial_\mu \chi_l \partial^\mu \chi_l, \quad (6)$$

where  $N_p = N_f(N_f - 1)/2$  is the number of different off-diagonal fields,  $\chi_{ij}$ . It is clear from the kinetic term why we chose the normalization of  $H$  in (5). Similarly the double trace potential is given by

<sup>2</sup>This question has recently been addressed in other two field models of inflation in [19–22].

$$v(\text{Tr}H^\dagger H)^2 = \frac{v}{4}\phi^4 + \frac{v}{4}\sum_l^{N_p}\chi_l^4 + \frac{v}{2}\phi^2\sum_l^{N_p}\chi_l^2 + \frac{v}{2}\sum_{l>k}^{N_p}\chi_l^2\chi_k^2. \quad (7)$$

For a general complex matrix  $H$  the potential can be fully written in terms of the structure constant of  $U(N_f)$ , see Appendix B of Ref. [23]. We will however here use the following grouping of terms for the single trace potential:

$$\begin{aligned} u\text{Tr}H^\dagger HH^\dagger H \\ = \frac{u}{4N_f}\phi^4 + \frac{3u}{2N_f}\phi^2\sum_l^{N_p}\chi_l^2 + \frac{3u}{\sqrt{2N_f}}\phi\sum_{i<j<k}^{N_f}\chi_{ij}\chi_{jk}\chi_{ki} \\ + \frac{u}{8}\sum_l^{N_p}\chi_l^4 + \frac{u}{8}\sum_{i\neq jk>i}^{N_f}\chi_{ij}^2\chi_{jk}^2 + \frac{u}{16}\sum_{i,j,k,l}^{N_f}\chi_{ij}\chi_{jk}\chi_{kl}\chi_{li}. \end{aligned} \quad (8)$$

It is worth noting that there is no cubic term for  $\phi$ . This is a consequence of the fact that  $\phi$  appears only on the diagonal of  $H$ .

At first glance this model seems overly complicated, however a short motivation why this model is relevant for discussion will be given here. A central parameter in the study for parametric resonance is the ratio of the quartic inflaton coupling ( $\lambda$ ) to the portal coupling ( $g^2$ ). A study similar to the present can be given in a toy model with an inflaton and a scalar field coupled to the inflaton. In these toy models the relevant parameter ( $\lambda/g^2$ ) will need to be fixed arbitrarily, see e.g. [7,9,20,21]. This is in contrast to the model of this paper. With the model given by (1) this ratio is given by the model itself and is therefore not an arbitrarily chosen number. Furthermore, as was shown in [2] the running of the couplings follow that of the gauge coupling (in [2] called  $\alpha_g$ ) along the UV unidimensional stable trajectory. This implies that the ratio  $\lambda/g^2$  stays constant even including running away from the UV fixed point. This is a remarkable fact that solidifies our future choice for this ratio to be that at the UV fixed point. This is *a priori* not a feature of a toy model, hence the model in (1) is an interesting scenario.

### III. RECAP OF RESULTS ON INFLATION

We will in this section recap some of the results of Ref. [3] as this is the inflationary scenario we have in mind for our investigation. Inflation is driven by the diagonal element of  $H$  where all diagonal elements are taken to be the same [2]. The inflationary effective potential can be derived from (1) with the couplings given by (2) and (3),

$$V(\phi) = \frac{\lambda\phi^4}{4(1+W(\phi))} \left( \frac{W(\phi)}{W(\mu_0)} \right)^{\frac{18}{135}}, \quad (9)$$

where  $\lambda = v^* + \frac{u^*}{N_f}$  and  $W(\cdot)$  is related to the product logarithm, see Ref. [3] for details. This is a  $\phi^4$  theory including renormalization of the inflaton operator expanded near the UV fixed point.

The potential in Eq. (9) is valid for a large range in  $\phi$ . However, for the study of parametric resonance small field values are considered as this minimum of the potential is located here. This means that for this analysis the potential will be approximated by  $\lambda\phi^4$ . This full  $\phi$  dependence is included for completeness.

It was shown that this model can provide a viable scenario for large field inflation. The inflationary slow-roll condition ceases to be satisfied and thus quasiexponential expansion stops at a field value

$$\phi_{\text{end}} = \sqrt{\left(4 - \frac{16}{19}\delta\right)\left(3 - \frac{16}{19}\delta\right)} M_p \approx \sqrt{12} M_p. \quad (10)$$

The phenomenological predictions for this model, for a very small  $\delta$ , lie just outside the Planck 2015  $2\sigma$  contours for the tensor-to-scalar ratio and scalar spectral index. It was noted that in this perturbative regime a very large number of flavors was needed to produce the measured amplitude of curvature perturbations by the inflaton fluctuations alone. However this number drops rapidly as the perturbative parameter  $\delta$  is pushed close to and beyond the radius of convergence of the underlying model.

The inflationary phase will quasiexponentially redshift the wavelength of any fluctuations existing before the onset of inflation, and will produce a homogeneous inflaton condensate. Once the field value of this condensate decreases to below the value given by (10), accelerated expansion of space ends and the reheating period begins. We will be working in terms of the usual metric

$$ds^2 = dt^2 - a(t)^2 d\mathbf{x}^2 \quad (11)$$

of space-time, where  $t$  is physical time and  $\mathbf{x}$  are the Euclidean comoving coordinates of the expanding space. It will often be useful to work in terms of conformal time  $\eta$  defined via  $dt = a d\eta$ . Since we are interested in the period of reheating, we will normalize the scale factor to be  $a(t_R) = 1$  at the time  $t_R$  corresponding to the beginning of reheating.

### IV. PARAMETRIC RESONANCE

We will here discuss parametric resonance of the inflaton and the off-diagonal scalar fields. One could also investigate parametric production of fermionic fields; however this will be left for a later discussion.

At the end of the inflationary epoch and before significant production of any other field has happened, the equation of motion (EoM) for the homogeneous inflaton field is given by

$$\ddot{\phi} + 3H\dot{\phi} + \lambda\phi^3 = 0, \quad (12)$$

where  $H$  is the Hubble expansion parameter. It is obvious that the solution for  $\phi$  will correspond to damped oscillation. The expansion of space can be factored out by introducing a rescaled field  $\tilde{\phi} \equiv a\phi$  and working in terms of conformal time. In terms of this field the solution is, as noted in [24], oscillatory, however not sinusoidal but proportional to the elliptic cosine,  $cn(x)$ , where  $x$  is a rescaled dimensionless conformal time which will be defined below. The equation of state of these oscillations is (upon time averaging) that of radiation. Hence the amplitude, which is asymptotically given by

$$\tilde{\phi} = a \frac{1}{\sqrt{t}} \left( \frac{3M_P^2}{8\pi\lambda} \right)^{\frac{1}{4}}, \quad (13)$$

is constant.

The EoM for the  $ij$  off-diagonal component is given by

$$0 = \ddot{\chi}_{ij} + 3H\dot{\chi}_{ij} + g^2\phi^2\chi_{ij} - a^{-2}\nabla^2\chi_{ij} + \frac{3u}{\sqrt{2N_f}}\phi \sum_k^{N_f} \chi_{ik}\chi_{kj} + \mathcal{O}(\chi^3), \quad (14)$$

where  $\nabla$  stands for the gradient operator with respect to the comoving spatial coordinates. Here

$$g^2 = v + \frac{3u}{N_f}. \quad (15)$$

Note that first line is leading order in  $\chi \sim 0$ , and anything beyond this is subleading. In our study of parametric resonance we consider the first three terms only, and the rest we regard as backreactions in the next section. The first line is a linear equation for  $\chi_{ij}$ , which we will solve for each Fourier mode independently.

Fourier transforming this linear EoM and rescaling the field by  $a\chi = X$  yields<sup>3</sup>

$$\tilde{X}_k'' + \left( \kappa^2 + \frac{g^2}{\lambda} cn^2\left(x, \frac{1}{\sqrt{2}}\right) - \frac{a''}{a} \right) \tilde{X}_k = 0, \quad (16)$$

where we have dropped the  $ij$  indices since the equation is identical for each component as long as we do not include mixing terms from the higher orders in  $\chi$ . We have defined

<sup>3</sup>We are neglecting terms in the equation of motion for  $X_k$  containing the metric perturbations. As discussed in Chap. 19 of the review article [25] this is justified as long as the energy density in the entropy field is smaller than that of the inflaton field. We are not the first to use this approximation. It is the basis of the *curvaton scenario* [26] and related scenarios such as the *new ekpyrotic model* [27].

$$\kappa^2 = \frac{k^2}{\lambda\tilde{\phi}^2} \quad (17)$$

and primes are derivatives with respect to the dimensionless conformal time

$$x \equiv \sqrt{\lambda}\tilde{\phi}\eta. \quad (18)$$

In (16) we have signified the Fourier component by a tilde, however tildes will be omitted from now on. The term  $\frac{a''}{a}$  is zero during the phase of parametric resonance as can be checked with the Friedman equation.

We will now approximate the elliptic cosine by the first term of a cosine expansion [24] as follows:

$$cn\left(x, \frac{1}{\sqrt{2}}\right) \simeq \frac{8\sqrt{2}\pi}{T} \frac{e^{-\frac{\pi}{2}}}{1 + e^{-\frac{\pi}{2}}} \cos\left(\frac{2\pi}{T}x\right), \quad (19)$$

where  $T$  is given by the complete elliptic integral

$$T = 4K\left(\frac{1}{\sqrt{2}}\right) = 4F\left(\frac{\pi}{2} \middle| \frac{1}{2}\right).$$

Here

$$F(\theta|m^2) = \int_0^\theta \frac{d\phi}{\sqrt{1 - m^2 \sin^2(\phi)}}$$

is the elliptic integral. Numerically we have  $\beta \equiv \frac{2\pi}{T} \simeq 0.8472$ .

This series expansion is not ideal for our situation. It might be the best behaved in terms of convergence, however for approximate use the amplitude of the first term is too small. As an approximation to the elliptic cosine we will use

$$cn\left(x, \frac{1}{\sqrt{2}}\right) \simeq \cos(\beta x). \quad (20)$$

With this we can rewrite (16) in terms of the rescaled dimensionless conformal time,  $y \equiv \beta x$  (which we normalize to be  $y = 0$  at the beginning of the reheating period, i.e.  $t = t_R$ ) as

$$X_k'' + \beta^{-2} \left( \kappa^2 + \frac{g^2}{2\lambda} + 2\frac{g^2}{4\lambda} \cos(2y) \right) X_k = 0, \quad (21)$$

where the derivative is with respect to  $y$ .

Written on this form, the equation of motion for  $\chi$  takes the form of a Mathieu equation with parameters  $q = \frac{g^2}{4\beta^2\lambda}$  and  $A = \beta^{-2}(\kappa^2 + 2q)$  (see e.g. [28]). The Mathieu equation has a solution given by



$$X(y) = A_1 \exp(\mu_k y) P_1(y, k) + A_2 \exp(-\mu_k y) P_2(y, k), \quad (22)$$

that is, a growing and decaying exponential solution, with periodic behavior captured by  $P_1$  and  $P_2$ . Here  $\mu_k$  is the Mathieu characteristic exponent, and the two periodic functions have amplitude 1 and the frequency which is given by the frequency of the inflaton condensate, i.e. independent of  $k$  (see [28,29]).

The characteristic exponent  $\mu_k$  is in general a complex number. However, for some parameters it has a real part called the Floquet index. Given our model parameters

$$\frac{g^2}{\lambda} = \frac{\alpha_v^* + 3\alpha_u^*}{\alpha_v^* + \alpha_u^*} \simeq 7.4, \quad (23)$$

we present the Floquet index in Fig. 1.

Before discussing the parametric amplification of  $\chi$  after inflation, we must determine the initial conditions for  $\chi(y)$  at the end of the inflationary phase. The fluctuation modes of the rescaled variables  $X$  (which are the canonical variables) begin in their quantum vacuum state. We now need to investigate whether or not fluctuation in  $\chi$  are squeezed during inflation on super-Hubble scales (on sub-Hubble scales the  $k^2$  term prevents squeezing in the same way that the corresponding term prevents the squeezing of inflaton fluctuations). Squeezing of the  $X$  fluctuations will transform the initial vacuum spectrum into a scale-invariant one in the same way that it leads to a scale-invariant spectrum of super-Hubble inflaton fluctuations. To check whether there is squeezing of the  $X$  perturbations during inflation we compare the neglected  $\frac{a''}{a}$  term from (16) to the induced mass term  $\frac{g^2}{\lambda}$ . In terms of regular time the condition for squeezing becomes

$$2H^2 > g^2 \frac{\tilde{\phi}^2}{a^2} = g^2 \phi^2. \quad (24)$$

Expressing the value of  $H$  during slow-roll inflation in terms of the field  $\phi$  show us that the squeezing condition is

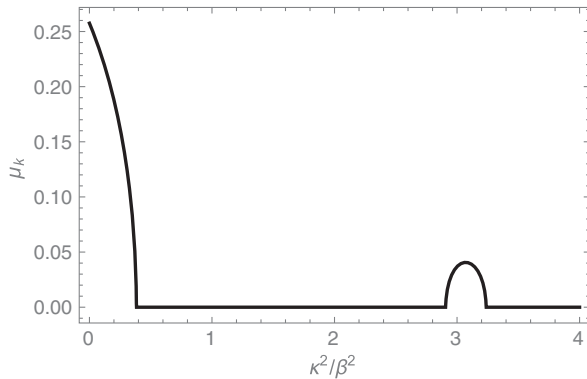


FIG. 1. Positive real part of the Mathieu characteristic exponent for  $q = \frac{g^2}{4\beta^2\lambda}$  and  $A = \beta^{-2}(\kappa^2 + 2q)$ .

satisfied throughout the period of inflation. Hence the spectrum of the  $X$  perturbations will be scale invariant at the beginning of the reheating period ( $t = t_R$ ), i.e.

$$X_k(t_R) \simeq H_I k^{-3/2} P_1(y=0, k), \quad (25)$$

where  $H_I$  is the value of  $H$  during inflation (more precisely when the scales of interest exit the Hubble radius during inflation). In the following we will not make a difference between  $H_I$  and the Hubble expansion rate at the end of the inflationary phase, i.e.  $H(t_R)$ .

During the preheating phase, the above value of  $X$  is exponentially amplified, yielding

$$X_k \simeq H_I k^{-3/2} \exp(\mu_k y) P_1(y, k). \quad (26)$$

Since we have normalized the scale factor to be  $a(t_R) = 1$ , and since we can ignore the growth of  $a(t)$  during the initial preheating period, we can identify  $X$  and  $\chi$ .

The fluctuations of  $\chi_k$  computed above yield the entropy fluctuations generated in our model. In order to compute the induced curvature fluctuations, we need the background value of the  $\chi$  field. In the case of two field inflationary models with scalar fields having a classical background, it is clear how to identify the background value of the entropy field. In the case of a model like the one we are considering, in which there is no classical zero mode of the entropy field, the situation is more complicated. Working strictly at first order in perturbation theory there is no background, and hence there will be no induced curvature fluctuations. However, this is clearly not the correct result, since if we were to argue in this way then cosmic string formation in an early universe phase transition would never lead to curvature fluctuations on super-Hubble scales, and it is well known that such curvature perturbations are formed (see e.g. [30] for reviews on cosmic strings and structure formation). A way to address this issue was recently suggested in [20–22]: each  $k$  mode of the fluctuations lives in an *effective background*  $\chi_{ij}^{\text{eff}}(k)$  which is generated by all perturbation modes with smaller wavenumber.

The effective background is given by<sup>4</sup>

$$\chi_{ij}^{\text{eff}}(k) = \left( \int_0^k d^3 k' |X_{k'}|^2 \right)^{1/2}. \quad (27)$$

To see this, we begin from the expression for the contribution of long (i.e. longer than  $k^{-1}$ ) modes to the  $X$  field at a fixed point  $x$  in space

<sup>4</sup>To further justify this, imagine that  $\chi$  contains fluctuations with two Fourier modes, a mode  $k$  we are interested in, and a longer wavelength mode  $k'$ . The mode  $k$  can locally be viewed as a mode which fluctuates not about 0, but about the local value of the  $k'$  mode.

$$X(x) = V^{1/2} \int_0^k d^3 k' X_{k'} e^{ik'x}, \quad (28)$$

in terms of the Fourier modes. Here,  $V$  is the cutoff spatial volume which we introduced such that the Fourier modes have the mass dimension of a harmonic oscillator. Without loss of generality we can take the point  $x$  to be  $x = 0$ . We now consider the expectation value of the square of the absolute value of (28)

$$\langle |X(0)|^2 \rangle = V \int_0^k d^3 k' \int_0^k d^3 k'' \langle X_{k'} X_{k''}^* \rangle. \quad (29)$$

Since the Fourier modes are uncorrelated we have

$$\langle X_{k'} X_{k''}^* \rangle \delta^3(k' - k'') V^{-1} |X_{k'}|^2. \quad (30)$$

Inserting (30) into (29) then yields (27).

We are interested in infrared modes  $k$  which lie in the instability band of the Mathieu equation. From Fig. 1 we see that for these values of  $k$  the infrared modes  $k'$  which appear inside the integral also lie in the instability band, and that we can approximate the Floquet exponent by a constant  $\mu_{k'} = \mu$ . Inserting (26) into (27) we see that there is a potential logarithmic infrared divergence of the integral. The periodic function  $P(y, k')$  appears in the integrand in quadratic form and hence does not eliminate the infrared divergence. There is, however, an infrared cutoff: the form (26) does not apply for modes which are outside the Hubble radius at the beginning of inflation. Therefore the integral can be estimated by

$$\chi_{ij}^{\text{eff}}(k) \sim H_I \exp(\mu y) \left( \ln \left( \frac{k}{k_{\min}} \right) \right)^{1/2} P_1(y, k), \quad (31)$$

where  $k_{\min}$  is the value of  $k$  which corresponds to Hubble radius crossing at the beginning of the period of inflation. The logarithm is given by  $N_I$ , the number of  $e$ -foldings of inflation.

## V. BACKREACTION EFFECTS

In the previous section we studied preheating in the UV-safe theory we introduced in the current paper neglecting any backreaction mechanism. Having done that we observed that exponential amplification sets in, and that the induced curvature fluctuations might have potentially dangerous consequences for the theory. However it is crucial to consider backreaction effects since they will eventually terminate the resonance. Backreaction effects are nonlinear and are often studied numerically. However, for the questions of large-scale curvature fluctuations, an analytical analysis is preferable (see [31] for an initial study of backreaction effects during preheating in a two field inflation model).

Here we will consider two kinds of backreaction effects:

- (i) The effect of produced  $\chi$  particles on the evolution of the inflaton field.
- (ii) The contribution of amplified modes on the effective mass for  $\chi$  field fluctuations.

Other backreaction effects are studied in [21,24] but are not relevant in our case.

### A. Effects on the evolution of the inflation field

The leading order term in  $\chi$ , for small  $\chi$ , in the inflaton action is

$$\frac{g^2}{2} \phi^2 \sum_l^{N_p} \chi_l^2, \quad (32)$$

where the index  $l$  runs over all of the  $\chi$  fields. This must remain subdominant to the main interaction term in the inflaton action, which is

$$\frac{\lambda}{4} \phi^4. \quad (33)$$

This leads to the condition

$$\sum_l^{N_p} \langle \chi_l \rangle_{\text{eff}}^2 < \frac{\lambda}{2g^2} \phi_{\text{end}}^2, \quad (34)$$

which needs to be satisfied in order to justify neglecting backreaction effects. In the case when all preheating fields are excited equally, which we show is the case, we get

$$\langle \chi_l \rangle_{\text{eff}}^2 < \frac{\lambda}{2g^2 N_p} \phi_{\text{end}}^2. \quad (35)$$

Naturally, the more fields we have, the less each must be excited to interfere. Note that all  $\chi$  modes which are excited contribute to the left-hand side of (35), and hence the expression is proportional to  $N_I$ .

### B. Contribution to the effective mass of the preheating field fluctuation

Now we focus on the mass term in the equation of motion for one of the  $\chi_k$  field modes. At linear order in  $\chi$ , the mass term comes from the coupling of  $\chi$  to the inflaton. This is the mass term which we have considered and which leads to the parametric instability, and its value is

$$g^2 \phi_{\text{end}}^2. \quad (36)$$

However, beyond linear order there is a contribution to the mass which comes from the interactions between all  $\chi$  fields. As is evident from (7) and (8) each  $\chi$  field couples quadratically to a fixed  $\chi$ . Assuming that all  $\chi$  fields are excited equally we get a contribution to the mass which is

$$\lambda' N_p \langle \chi \rangle_{\text{eff}}^2, \quad (37)$$

where we have taken into account that each mode of  $\chi$  which is excited contributes to the effective mass of the  $\chi$  field, and hence the above expression is proportional to  $N_I$ . In the above,  $\lambda'$  is a coupling constant made up of the constants appearing in (7) and (8). The condition, that backreaction can be neglected, then becomes

$$\langle \chi \rangle_{\text{eff}}^2 < \frac{g^2}{2\lambda' N_p} \phi_{\text{end}}^2. \quad (38)$$

Since  $\lambda'$  is of the same order of magnitude as  $\lambda$  we find that (using the value of  $g^2/\lambda$  which our model predicts) the first backreaction condition (35) is slightly stronger than the second one (38).

Note that when the parametric resonance stops all preheating fields have been excited equally as the sole nondemocratic couplings enter in the  $\chi^4$  terms relevant only for the second backreaction effect considered.

## VI. INDUCED CURVATURE PERTURBATIONS

Having shown that parametric resonance of the spectator scalar fields is efficient in this model, we move on to investigate the resulting amplification of the entropy fluctuations, which in turn leads to a contribution to the curvature perturbation with an exponentially growing amplitude.

Fluctuations in a spectator scalar field will induce a contribution to the curvature whenever the equation of state of the spectator field mode is different from that of the adiabatic mode. The magnitude of the induced curvature fluctuation is proportional to the energy density in the spectator field. Thus, a background value of the spectator field is required in order to obtain a growing curvature mode. If we are considering curvature fluctuations on a fixed scale  $k$ , we will use the effective background  $\chi$  field constructed earlier as this background. The conversion of entropy field fluctuations into curvature perturbations has been studied in many works (see e.g. [32,33] for some classic papers). We may also use the covariant formalism of [34] as applied to the study of preheating in [20]. The result of these analyses is that the induced curvature fluctuation is given by

$$\zeta_k \simeq \frac{H}{\dot{\phi}^2} \dot{\chi} S_k, \quad (39)$$

where  $S_k$  is the entropy field perturbation which, according to our analysis, is given by

$$S_k = H k^{-3/2} \exp(\mu y) P_1(y, k), \quad (40)$$

where  $P_1(y, k)$  is the same periodic function as in (26).

The expression for  $\dot{\chi}$  can be found using (31), yielding

$$\dot{\chi} = \beta \sqrt{\lambda} \Phi H \sqrt{N_I} \exp(\mu y) \frac{\partial P_1(y, k)}{\partial y}, \quad (41)$$

where  $\Phi$  is the amplitude of  $\phi$ , and where we have neglected the derivative of the exponential factor, as it carries a factor of  $\mu$  which is small compared to the order 1 frequency of  $P_1$ . Hence, we obtain

$$\zeta_k = \frac{H^3}{\dot{\phi}^2} \sqrt{\lambda} \beta \sqrt{N_I} \exp(2\mu y) k^{-3/2} \Phi P(y, k), \quad (42)$$

where  $P(y, k) = P_1 \frac{\partial P_1}{\partial y}$  is a periodic function. The most important feature of this result is exponential growth of curvature perturbation which is induced by the entropy perturbation (see also the Appendix of Ref. [20] for a more detailed derivation of this result).

Using this result we can evaluate the power spectrum of induced curvature perturbations, yielding

$$P_k = \frac{k^3}{2\pi^2} |\zeta_k|^2 \simeq \frac{H^6 \Phi^2}{\dot{\phi}^4} \exp(4\mu y) \frac{\beta^2 \lambda}{4\pi^2} N_I, \quad (43)$$

where we used  $|P|^2 \sim \frac{1}{2}$ . To estimate this expression we will use the fact that at the end of inflation kinetic energy is of the same order of magnitude as the potential energy. We also use  $\Phi = \phi_{\text{end}}$  and introduce the number  $\sigma$  via

$$\phi_{\text{end}} \equiv \sigma M_p. \quad (44)$$

We can use the result of the previous section on backreaction to yield an estimate for the value of  $\exp(4\mu y)$  when the resonance stops. As discussed in the previous section one can show the first backreaction effect shuts off the resonance before the second. Therefore using (35) for the time when the backreaction becomes important, we get

$$\exp(4\mu y) \sim \left( \frac{2g^2}{\lambda} \right)^{-2} \frac{M_p^4}{H^4} N_I^{-2} N_p^{-2} \sigma^4. \quad (45)$$

Inserting (45) and (44) into (43) and taking into account that the potential energy at the end of inflation is

$$V = \frac{\lambda}{4} \sigma^4 M_p^4, \quad (46)$$

we obtain our final result,

$$P_k \sim \frac{\beta^2 \sigma^2}{N_p^2 N_I} \left( \frac{g^2}{\lambda} \right)^{-2}, \quad (47)$$

for the power spectrum of the induced curvature fluctuations.



For our model with  $\frac{q^2}{\lambda} \simeq 7.4$  and  $\sigma^2 \simeq 12$  and  $\beta \simeq 1$  we get

$$P_k \sim \frac{1}{N_p^2 N_I}. \quad (48)$$

For this not to exceed the observed value with an amplitude of order  $10^{-10}$  we need a large number of flavors and/or a large number of  $e$ -foldings of inflation.

## VII. CONCLUSION

In this paper we have considered reheating in a class of asymptotically safe quantum field theories recently studied in [1,2]. These theories allow for an inflationary phase in the very early universe. Inflation ends with a period of reheating. Since the models contain many scalar fields which are intrinsically coupled to the inflaton there is the possibility of parametric resonance instability in the production of these fields, and the danger that the induced curvature fluctuations will become too large.

Our first result is that the parametric instability indeed arises, and hence that the energy transfer from the inflaton condensate to fluctuating fields is rapid.

Our second result concerns the demand that the curvature fluctuations induced by the parametrically amplified entropy modes do not exceed the upper observational bounds. We have seen that this puts a lower bound on the product  $N_p^2 N_I$ , where  $N_p$  is the number of scalar fields which the model of [1,2] contains, and  $N_I$  is the total number of  $e$ -foldings of the inflationary phase. The reason that the power spectrum of the induced curvature

fluctuations decreases as  $N_p^2$  is that backreaction effects turn off the parametric instability earlier as  $N_p$  increases. It is a linear effect in  $N_p$  on the fluctuation modes, and hence a quadratic effect in the power spectrum. The reason that the bound depends on  $N_I$  is that the energy density in the effective entropy field background (which determines the strength of the conversion of entropy to the adiabatic mode) is proportional to  $\sqrt{N_I}$ , and that as  $N_I$  increases the backreaction is shut off earlier due to more modes being super Hubble. The combination of these effects gives the net scaling of the power spectrum as  $N_I^{-1}$ .

Although we chose to investigate the parametric production of entropy fluctuations in a specific particle physics model (which was of interest to us for other reasons), our analysis can be extended rather straightforwardly to other multifield models. Features found in this paper will be present in other models as well, making our analysis more generally applicable.

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